A new Bound for an NP-hard Subclass of 3-SAT using Backdoors

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What can Backdoors be used for?

- To measure structure in SAT-instances
- Knowing a (small) backdoor can speed up the searching process (check $2^{|B|}$ assigns)

In general: worst-case runtime for SAT cannot be limited by backdoors

If no backdoor of a particular size exists?

We consider a subclass of SAT

$\Rightarrow$ There is a deterministic algorithm solely based on backdoors

OUTLINE

- Class $2^\star$-CNF
- Algorithm based on 2 kinds of backdoors
- Complexity of the algorithm

The class $2^\star$-CNF

- 2-SAT extended by clauses with 3 negative literals
  $(x_1 \vee x_2) \wedge (x_2 \vee \overline{x_3}) \wedge (\overline{x_4} \vee \overline{x_5}) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_6}) \ldots$
- $2^\star$-CNF is subclass of 3-SAT and MHF (Mixed Horn Formula)
- $2^\star$-CNF is NP-complete! Proof:
  - 3-colorability of graphs
  - reduction 3-SAT $\leq_p$ $2^\star$-CNF (paper)

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Idea of the Algorithm I

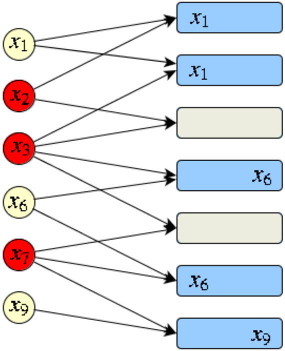
Search for Horn Backdoor

consider positive binary clauses

Remaining instance $F - B^+$ is Horn

Is there $|B^+| \leq d * |V|$?
($d \leftarrow 0.513$)

Yes? $\Rightarrow$ Solve F by using the Horn-Backdoor B^+



$B^+ :=$ red vars

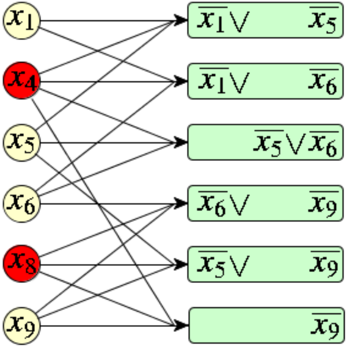
IDEA OF THE ALGORITHM II
SEARCH FOR BINARY BACKDOOR

consider negative ternary clauses

Remaining instance $F - B^-$ is binary

Is there $|B^-| \leq (1 - d) * |V|$?

Yes? $\Rightarrow$ Solve F by using the Binary-Backdoor B^-



$B^- :=$ red vars

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IDEA OF THE ALGORITHM III

If both answers were 'NO' ...
... the instance has to be unsatisfiable

- more than $d * |V|$ variables needed to cover all positive binary clauses
  $\Rightarrow$ # vars that must be set to *true* $> d * |V|$
- more than $(1 - d) * |V|$ variables needed to cover all negative ternary clauses
  $\Rightarrow$ # vars that must be *false* $> (1 - d) * |V|$

↯ This is not possible simultaneously!

Similar to Franco and Swaminathan for huge 3-SAT inst.

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Complexity of the Algorithm

The critical points

```
Function bd_solve(F)

  Choose minimum B⁺ ⊆ V to cover pos. binary clauses
! if(|B⁺| ≤ d * |V|) then
      return Solve F by using the Horn-Backdoor B⁺

  Choose minimum B⁻ ⊆ V to cover neg. ternary clauses
! if(|B⁻| ≤ (1 − d) * |V|) then
      return Solve F by using the Binary-Backdoor B⁻

return F Unsatisfiable
```

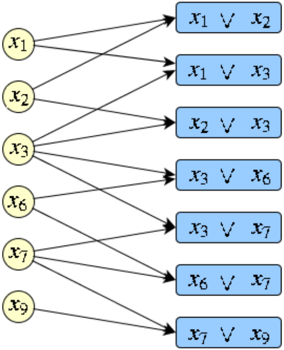
We need exact results!

PARAMETERIZED VERTEX COVER

TO COVER ALL POSITIVE BINARY CLAUSES

Graph $G = (V, E)$. Is there a subset $H \subseteq V$ with $\leq k$ vertices such that each edge in E has at least one of its endpoints in H?

Solvable in $O(k * |V| + 1.29^k)$ (Niedermeier)
$k := d|V|$



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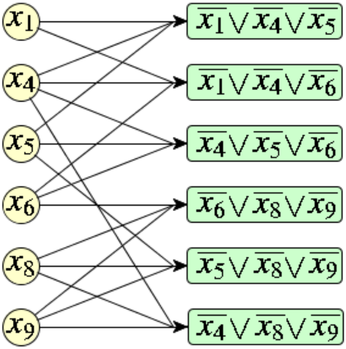
Parameterized 3-Hitting-Set

to cover all negative ternary clauses

Collection Q of subsets of size ≤ 3 of a finite set S.

Is there a subset $H \subseteq S$ with $\leq k$ elements such that H contains at least one element of each subset in Q?



$O(poly(n) * 2.0755^k)$
(Wahlström)
$k := (1 - d)|V|$

Overall Complexity

```
Function bd_solve(F)

Choose B⁺ ⊆ V to cover pos. 2-clauses              1.14^n
if(|B⁺| ≤ d * |V|) then
    return Solve F by the Horn-BD B⁺               1.427^n

Choose B⁻ ⊆ V to cover neg. 3-clauses              1.427^n
if(|B⁻| ≤ (1 − d) * |V|) then
    return Solve F by the Binary-BD B⁻             1.402^n

return F Unsatisfiable
```

Result: $O(poly(n) * 1.427^n)$

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Conclusion

- Class $2^{\star}$-CNF as subclass of 3-SAT and MHF
- Algorithm that uses the **existence** and **non-existence** of particular backdoors
- Slightly faster than known algorithms for 3-SAT and MHF

Thank you for your attention!