Beyond Unit Propagation in SAT Solving

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WHY SAT?

Bounded Model Checking

Verification

Plugin System

Automotive Product Configuration

Why SAT?

Bounded Model Checking

Verification

Plugin System

Automotive Product Configuration

SAT-Solver

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

OUTLINE

SAT Solving - DPLL

$$
\begin{aligned}
C_1 &= \{\overline{l_1} \vee l_6\} \\
C_2 &= \{\overline{l_6} \vee l_4\} \\
C_3 &= \{\overline{l_4} \vee \overline{l_6} \vee l_3\} \\
C_4 &= \{\overline{l_2} \vee l_7\}
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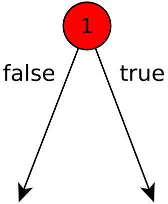
- Decisions



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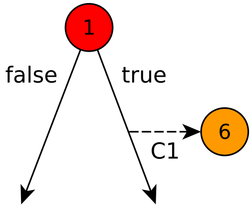

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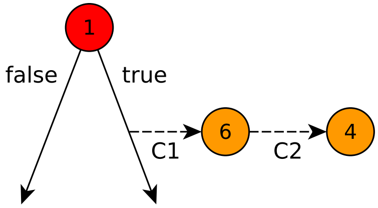

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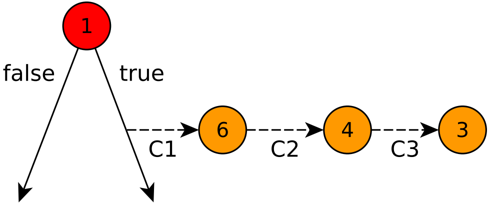

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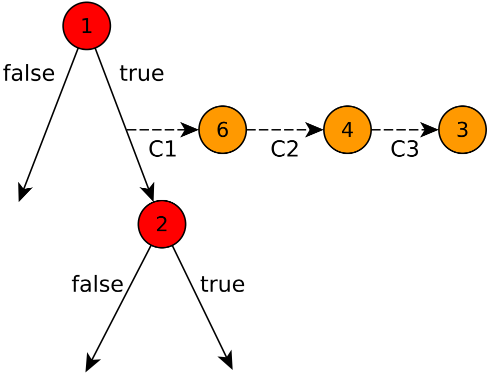

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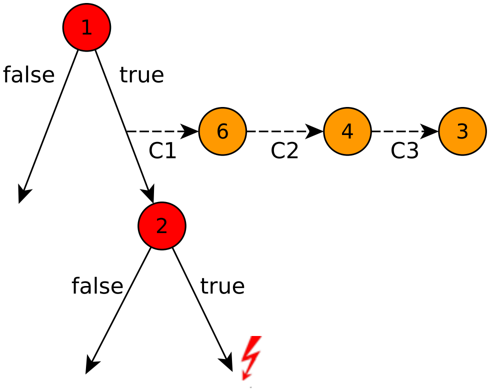
- Decisions
- Propagation of assignments
- Conflict analysis



Boolean Constraint Propagation

- search constitutes partial assignment π
- consider clauses that are unit under π

BOOLEAN CONSTRAINT PROPAGATION

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EXAMPLE (UNIT PROPAGATION)

$\pi = \overline{l_4}, \overline{l_5}, l_6 \ldots$

$C = \{l_4 \lor l_5 \lor l_8\}$ is *unit under* $\pi \Rightarrow l_8$ is implied

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- very efficient implementations
- $\geq 80\%$ of runtime

IDEA: DON'T WAIT FOR UNITS

EXAMPLE

$\pi = \overline{l_4}, \overline{l_5}, l_6 \ldots$

$C = \{l_4 \vee l_5 \vee l_1 \vee l_2 \vee l_3\}$

IDEA: DON'T WAIT FOR UNITS

EXAMPLE

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What can we do?

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$\pi = \overline{l_4}, \overline{l_5}, l_6 \dots$
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Clearly: one of l_1, l_2, l_3 has to be assigned

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e.g. $l_1 \Rightarrow l_7$, $l_2 \Rightarrow l_7$, $l_3 \Rightarrow l_7$

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l_7 can already be assigned!

Direct implications in CNF

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DIRECT IMPLICATIONS IN CNF

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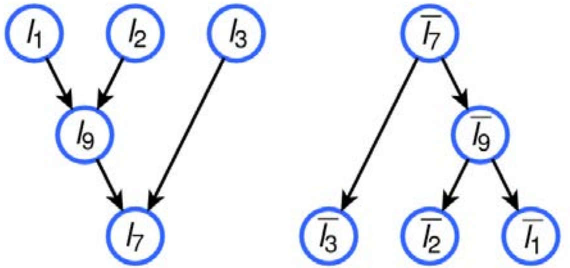

Implication graph induced by binary clauses

JUST A GRAPH PROBLEM

QUESTION?

Is there a common successor for a set of vertices?

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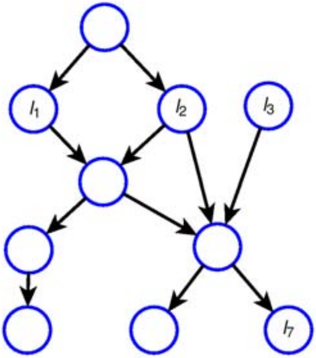

Just a graph problem

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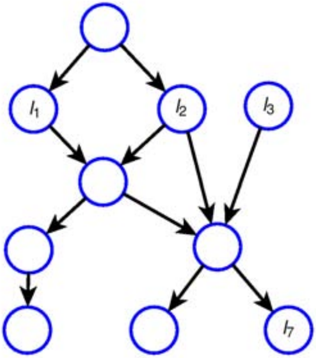

Trivial approach

Keep one bit for each pair of literals (l_i, l_j) which is set if l_i and l_j have a common successor

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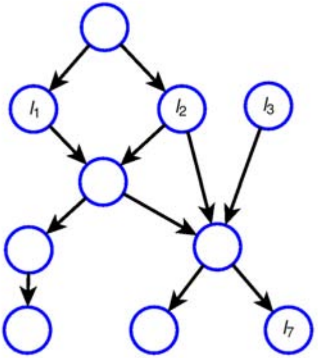


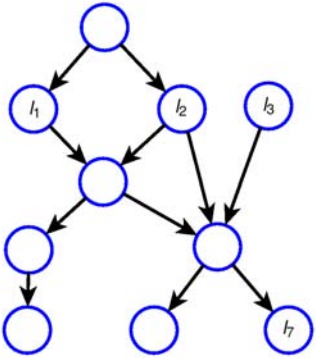
TRIVIAL APPROACH

Keep one bit for each pair of literals (l_i, l_j) which is set if l_i and l_j have a common successor

$\Downarrow$

Much too big!!

MATRIX COMPRESSION



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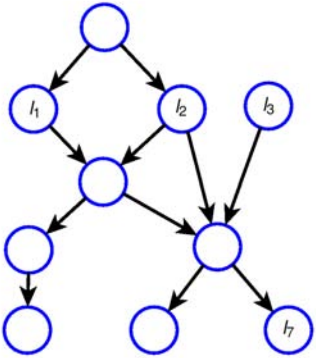

ONE IDEA:

In a DAG two vertices have common successor iff they reach same sink

$\Rightarrow$ store reachability of sinks

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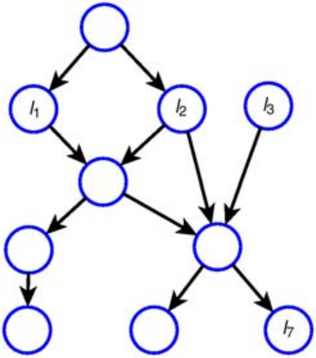

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...

...more compression techniques to make it work! [see paper!]

REVIEW

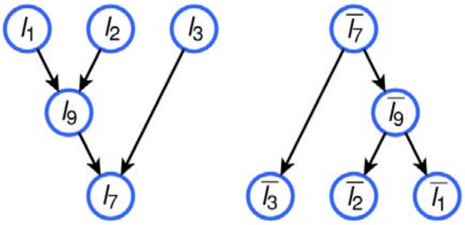
- matrices are still too big for some SAT instances
- adding many binary clauses requires matrix updates
- quite some work for implementation

S\ I	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	x							x									x	
2									x									
3									x									
4																	x	
5	x													x				
6			x															
7					x													
8		x											x			x		
9			x			x		x		x				x				
10	x																x	
11		x				x	x	x										
12					x							x	x			x		
13				x		x				x	x							
14	x		x														x	
15														x				
16		x					x										x	
17				x		x				x	x				x			
18																		x
19	x										x						x	x
20															x			
21				x								x				x	x	x

SINKS AND ROOTS

COMPLEMENTARY COMPONENTS

Flipped sinks of one component are roots in complementary component.



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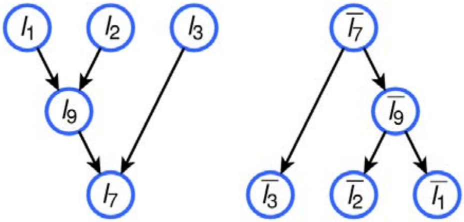

Still valid if complementary components are connected!

COLLECTING SINK TAGS

IDEA

Collect and cache information during normal unit propagation of binary clauses.

$$C_1 = \{\overline{l_1} \vee l_9\}$$
$$C_2 = \{l_2 \vee l_9\}$$
$$C_3 = \{l_3 \vee l_7\}$$
$$C_4 = \{l_9 \vee l_7\}$$

$\overline{l_7}$

Tag Table:

l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9
↓	↓	↓	↓	↓	↓	↓	↓	↓
-	-	-	-	-	-	-	-	-

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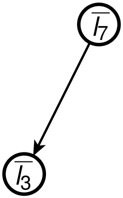

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-	-	l_7	-	-	-	-	-	-

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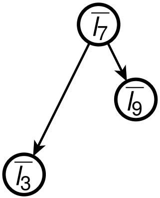

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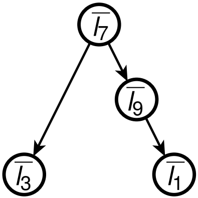

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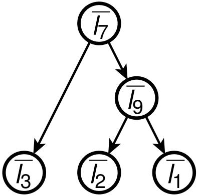

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Example

$$\pi = \overline{l_4}, \overline{l_5}, l_6 \ldots$$
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What can we do?

Using sink tags

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Example

$\pi = \overline{l_4}, \overline{l_5}, l_6 \ldots$
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What can we do? $\Rightarrow$ Simple table lookup

MATRIX VS. TAGS

- Tests on 500 hard instances of previous SAT competitions
- Timeout for each instance 1200 seconds

MATRIX VS. TAGS

- Tests on 500 hard instances of previous SAT competitions
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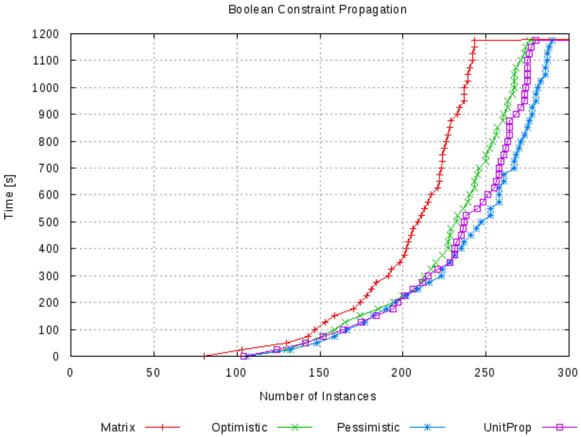
	Matrix		Tags	
	avg	max	avg	max
ext. Prop / Decisions [%]	63.24	1581.93	33.71	1340.64
Implied Binaries	16816.36	235042	9100.49	152728
Implied Units	101.48	2722	146.71	4386

RUNTIME


Boolean Constraint Propagation
Time [s]
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
0
50
100
150
200
250
300
Number of Instances
Matrix
Optimistic
Pessimistic
UnitProp

CONCLUSION

- Analysed Boolean Constraint Propagation
- Most quality improvement with matrix approach $\rightarrow$ but bad runtime
- Tag approach still clearly better than Unit Propagation $\rightarrow$ comes for free!!

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Thank you!